

Long-term perspectives of global economy - quantitative forecasting using the logistic growth curve

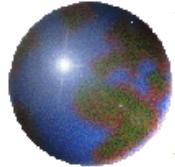
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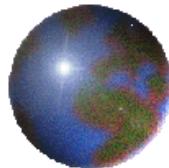
Wroclaw University

<http://prawo.uni.wroc.pl/~kwasnicki>



Logistic curve, s-shape curve, sigmoid curve

- ➊ different processes such as the diffusion of new technologies, creativity, population dynamics, product sales, corporate management, and stock prices can be modeled using the s-shaped logistic curves.



Logistic curve, s-shape curve, sigmoid curve

$$\frac{dy}{dt} = r \cdot y \left(1 - \frac{y}{K}\right)$$

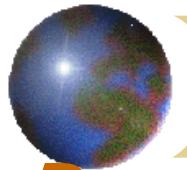
$$y = \frac{K}{1 + ae^{-bt}}$$

$$y = \frac{K}{1 + e^{-\frac{\ln(81)}{\Delta t}(t-t_m)}}$$

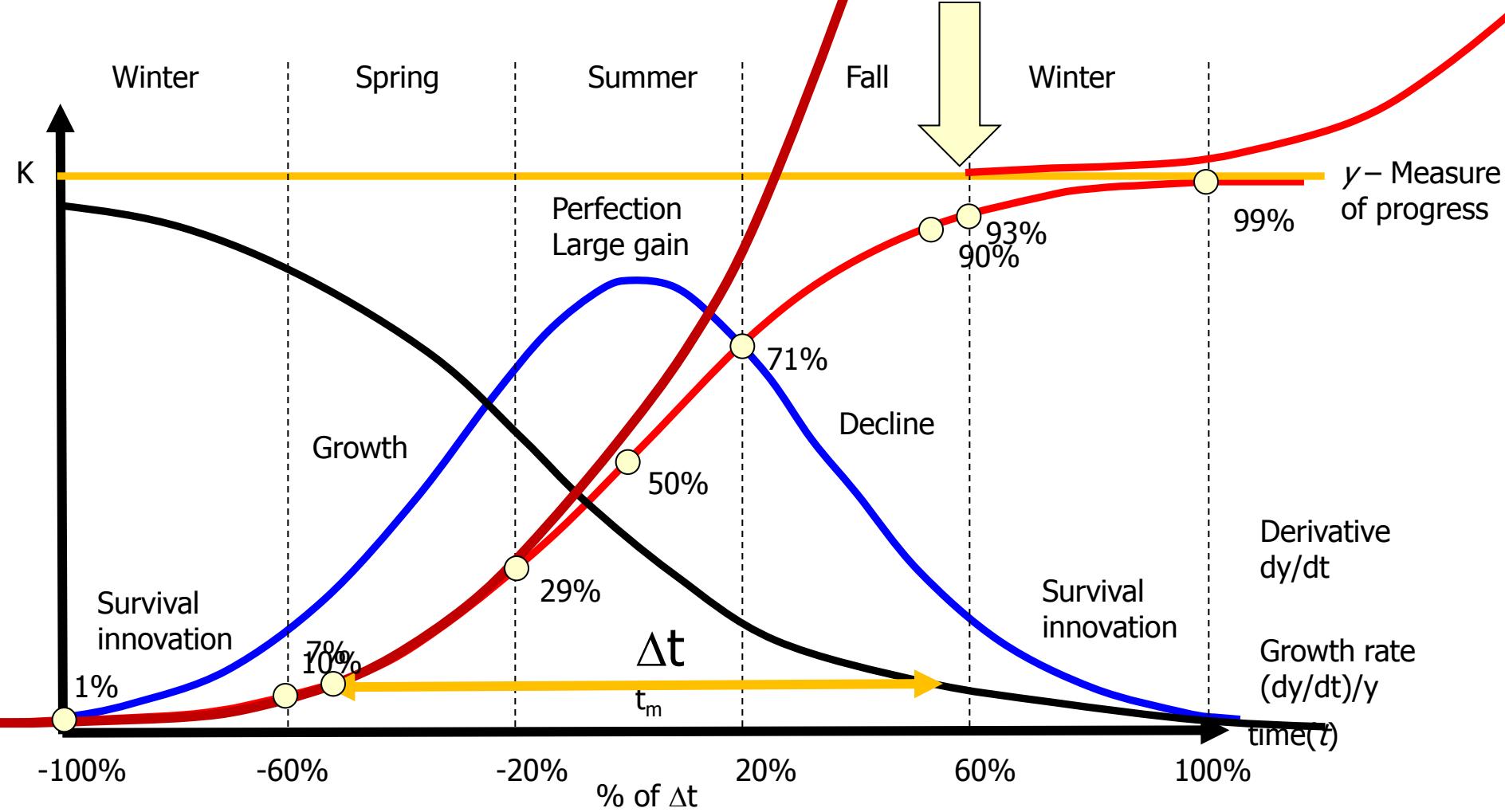
- ➊ y – measure of progress
- ➋ K – saturation level
(environment's capacity)

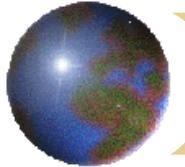
- ➌ Δt – the characteristic duration
of the curve i.e. the time
needed for y to grow from 10%
to 90% of K .
- ➍ t_m – the midpoint of the curve at
which 50% of K is reached.

$$\lim_{K \rightarrow \infty} y = \lim_{K \rightarrow \infty} \frac{K}{1 + ae^{-bt}} = Ae^{\gamma t}$$

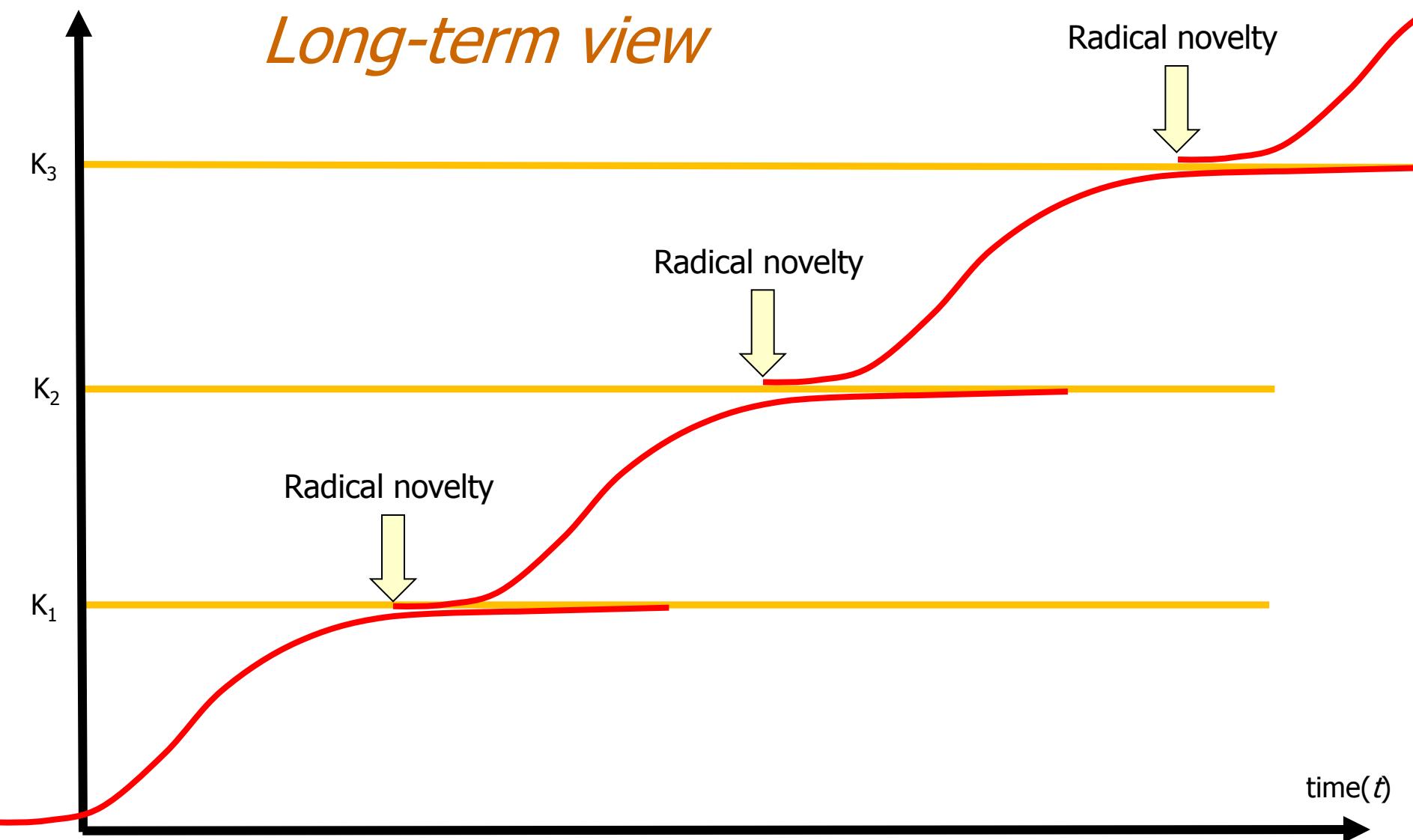


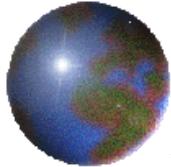
Development – qualitative view





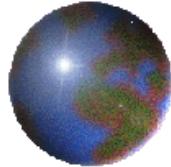
Long-term view





Goal:

- ◆ Having historical data on World Population Growth (1950-2008), World GDP and World GDP per capita (1950-2005) try to fit logistic curve and try to predict future values of these three characteristics.
- ◆ To fit the logistic curve (i.e., to identify its parameters K , Δt , and t_m) we ought to define the fitting criterion (i.e., identification criterion).
- ◆ Source of the historical data:
 - ▣ The Conference Board Total Economy Database
 - <http://www.conference-board.org/economics/database.cfm>

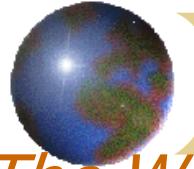


- ➊ Classical criterion is so called minial distance (mean square error):

$$Q_1 = \frac{1}{t_{max} - t_0} \sum_{t=t_0}^{t_{max}} (y^r(t) - y^m(t))^2$$

- ➋ We use also mean relative square error:

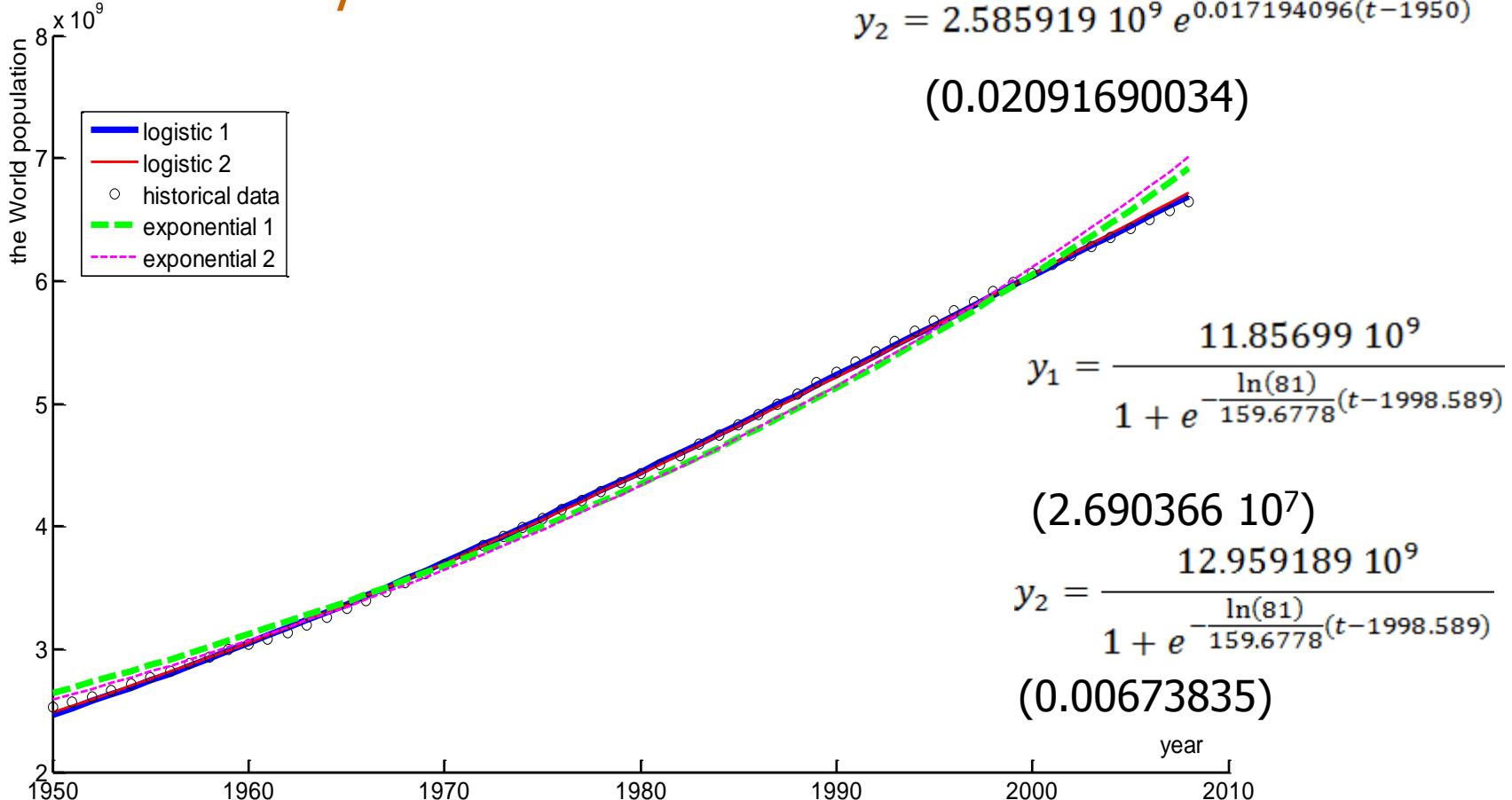
$$Q_2 = \frac{1}{t_{max} - t_0} \sum_{t=t_0}^{t_{max}} \left(\frac{y^r(t) - y^m(t)}{y^m(t)} \right)^2$$

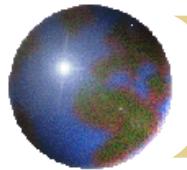


The World population

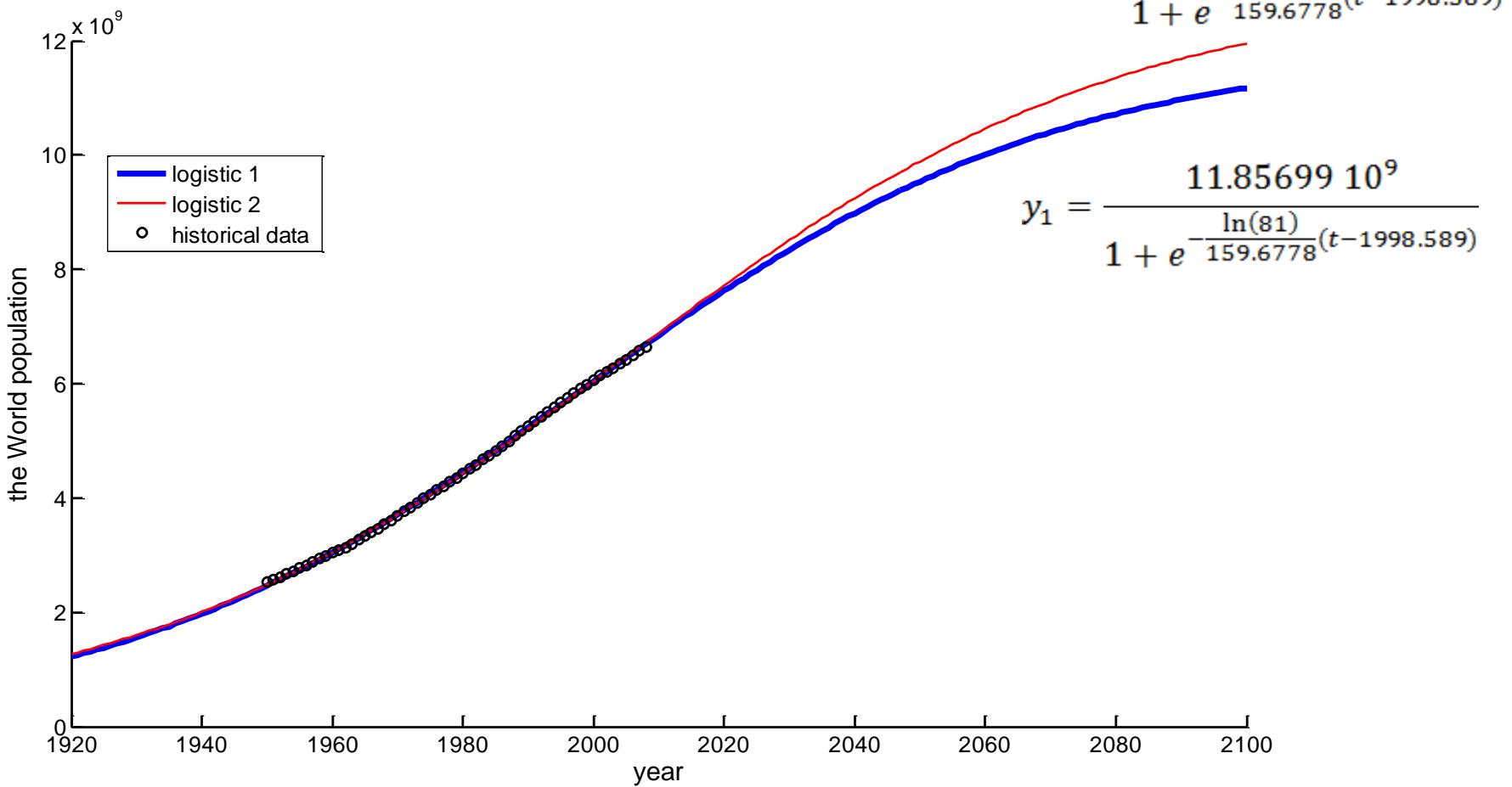
Quality of curve fitting

Identification period 1950-208



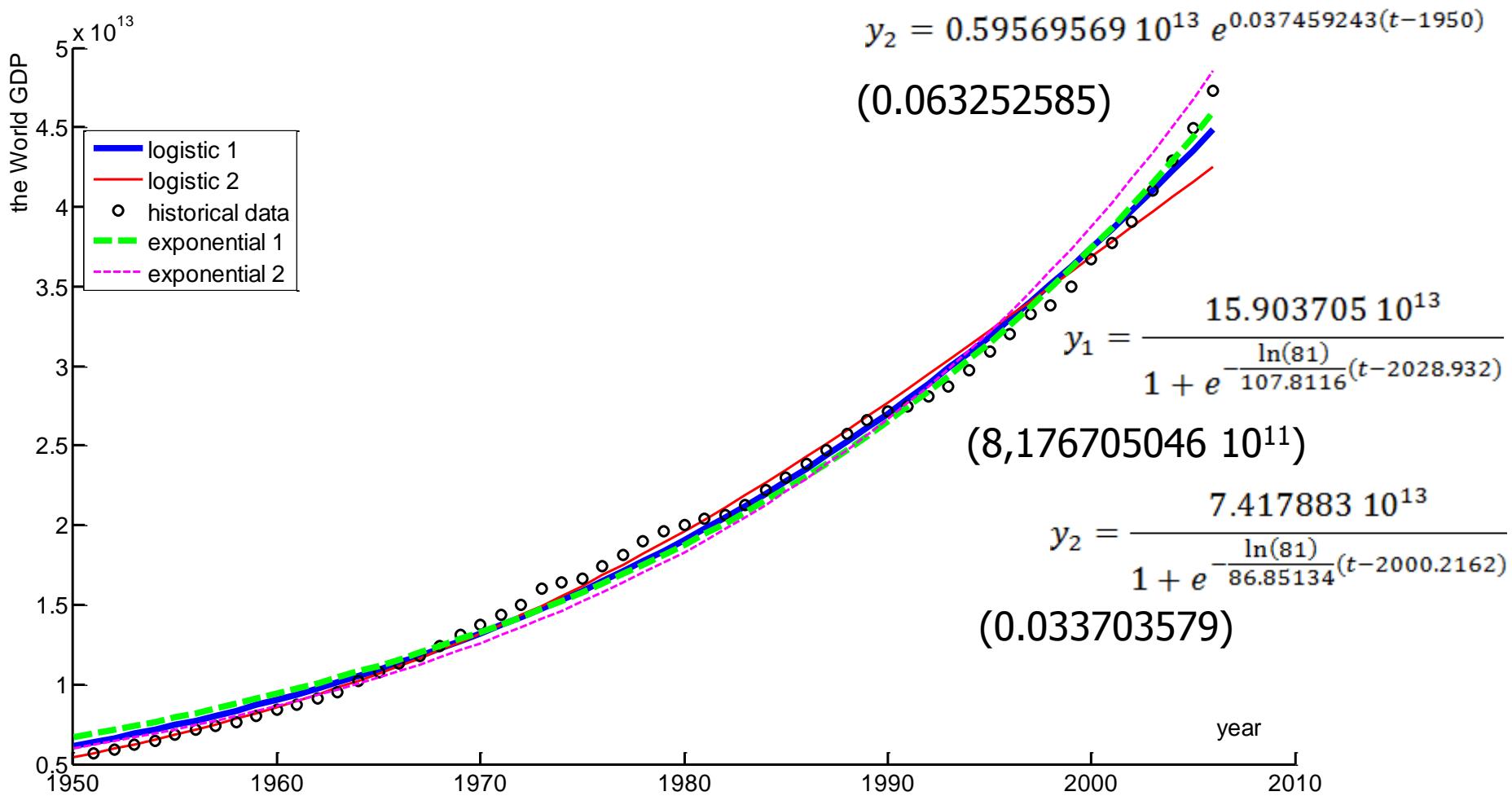


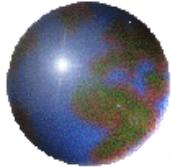
The World Population forecast (Identification period 1950-2008)



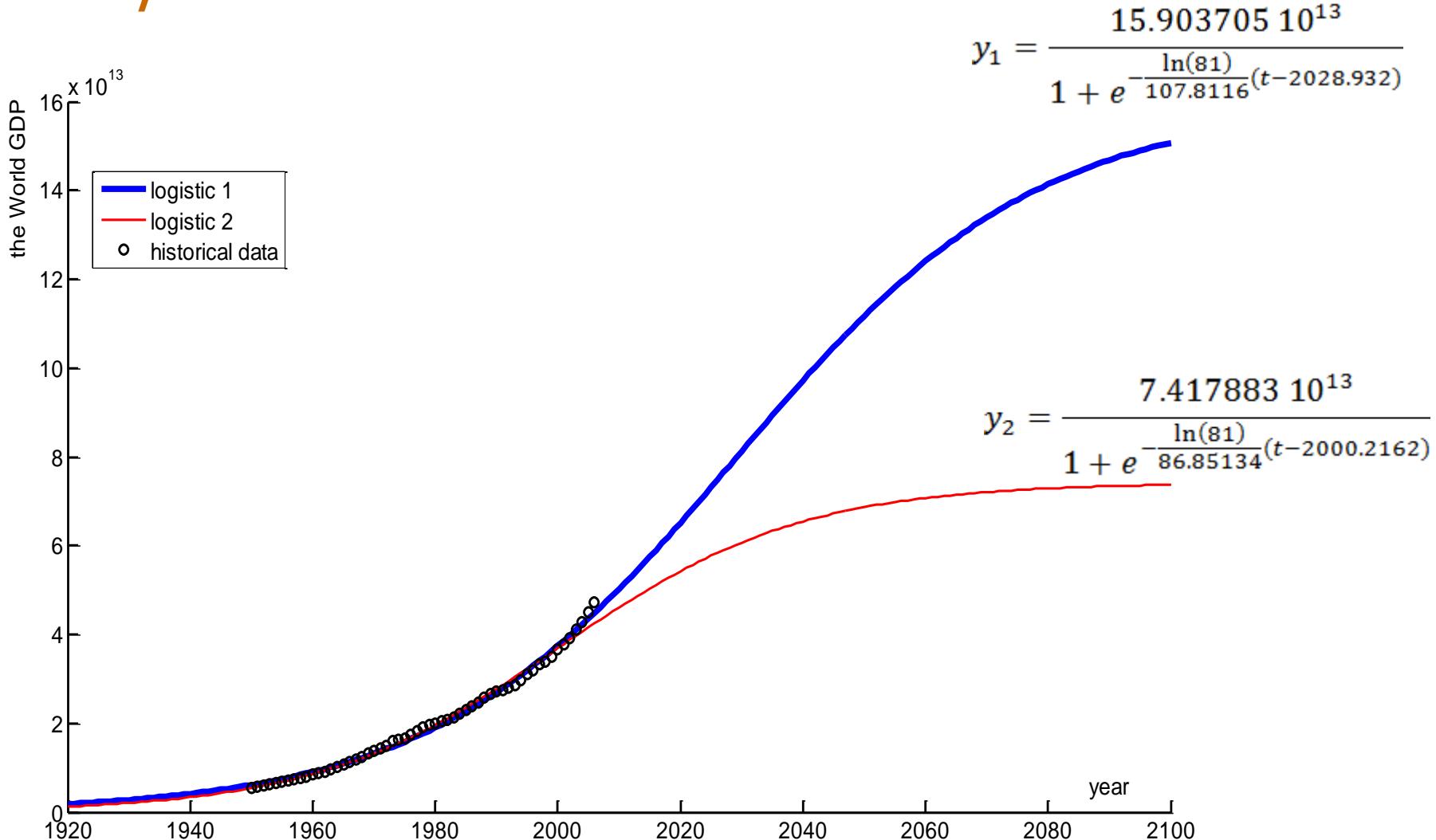


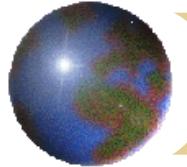
GDP curve fitting (identification period 1950-2006)



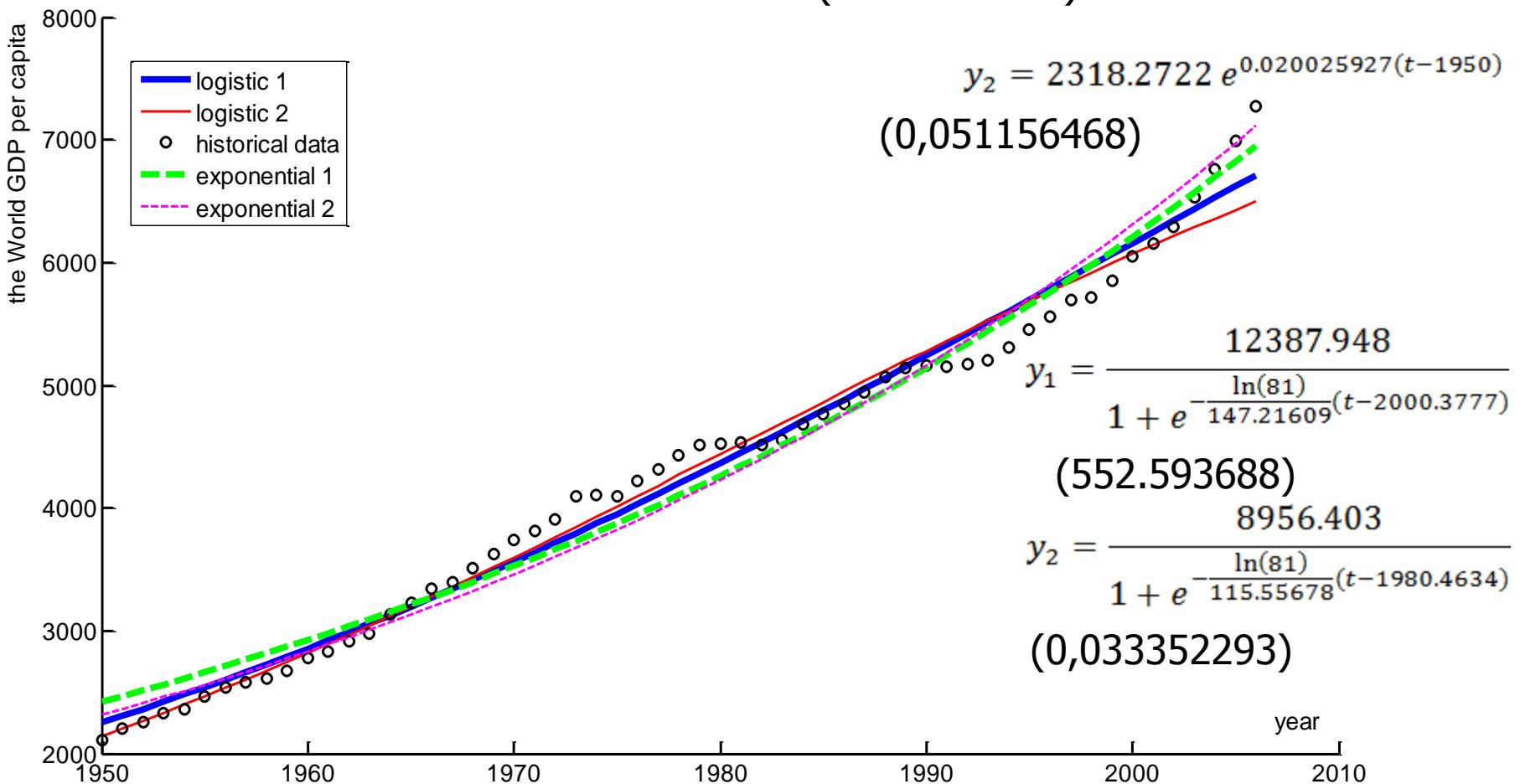


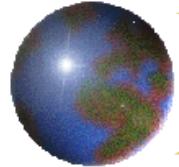
GDP prediction



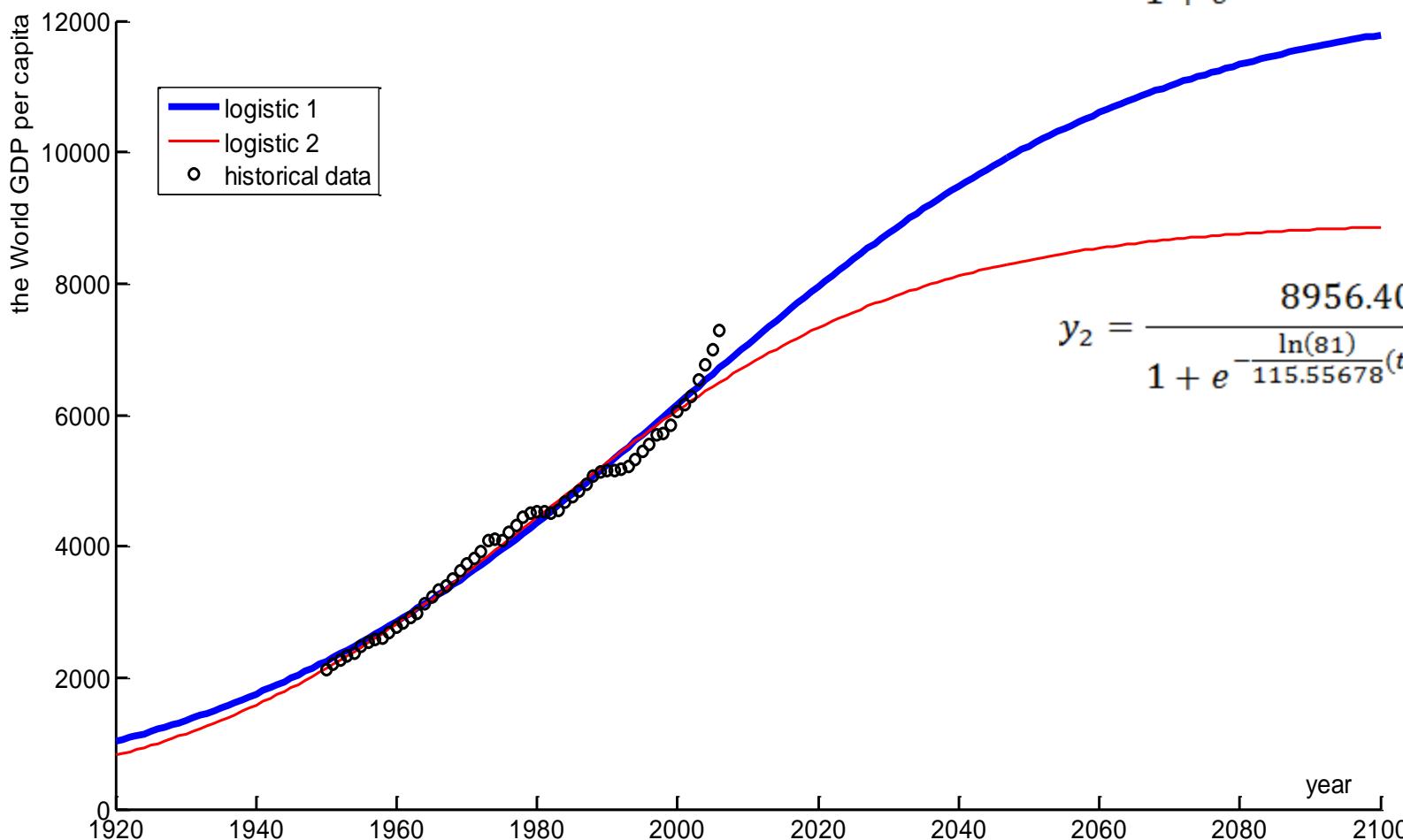


GDP per capita (curve fitting)



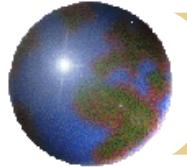


GDP per capita prediction (identification period 1950-2006)

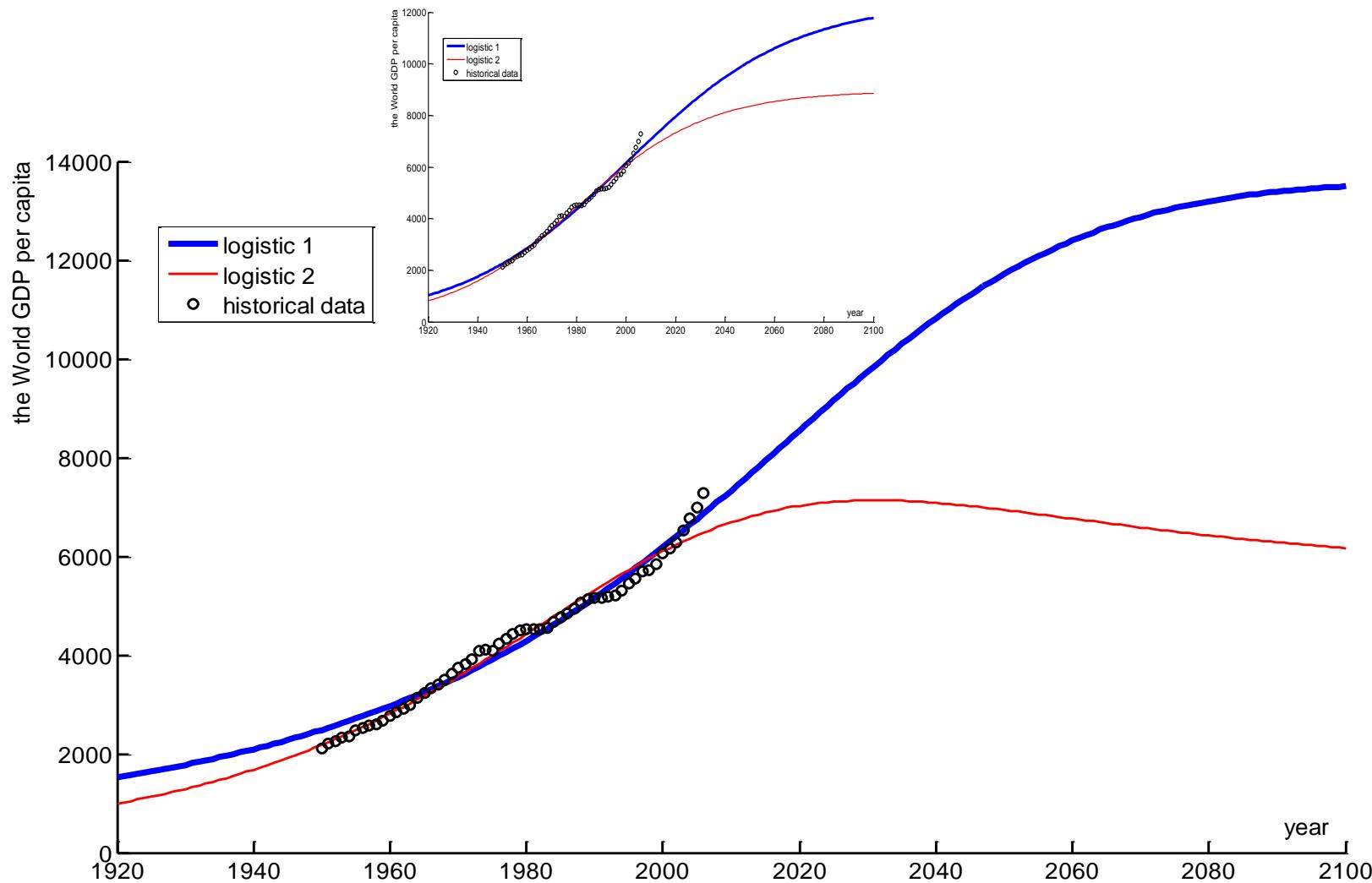


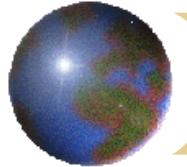
$$y_1 = \frac{12387.948}{1 + e^{-\frac{\ln(81)}{147.21609}(t - 2000.3777)}}$$

$$y_2 = \frac{8956.403}{1 + e^{-\frac{\ln(81)}{115.55678}(t - 1980.4634)}}$$

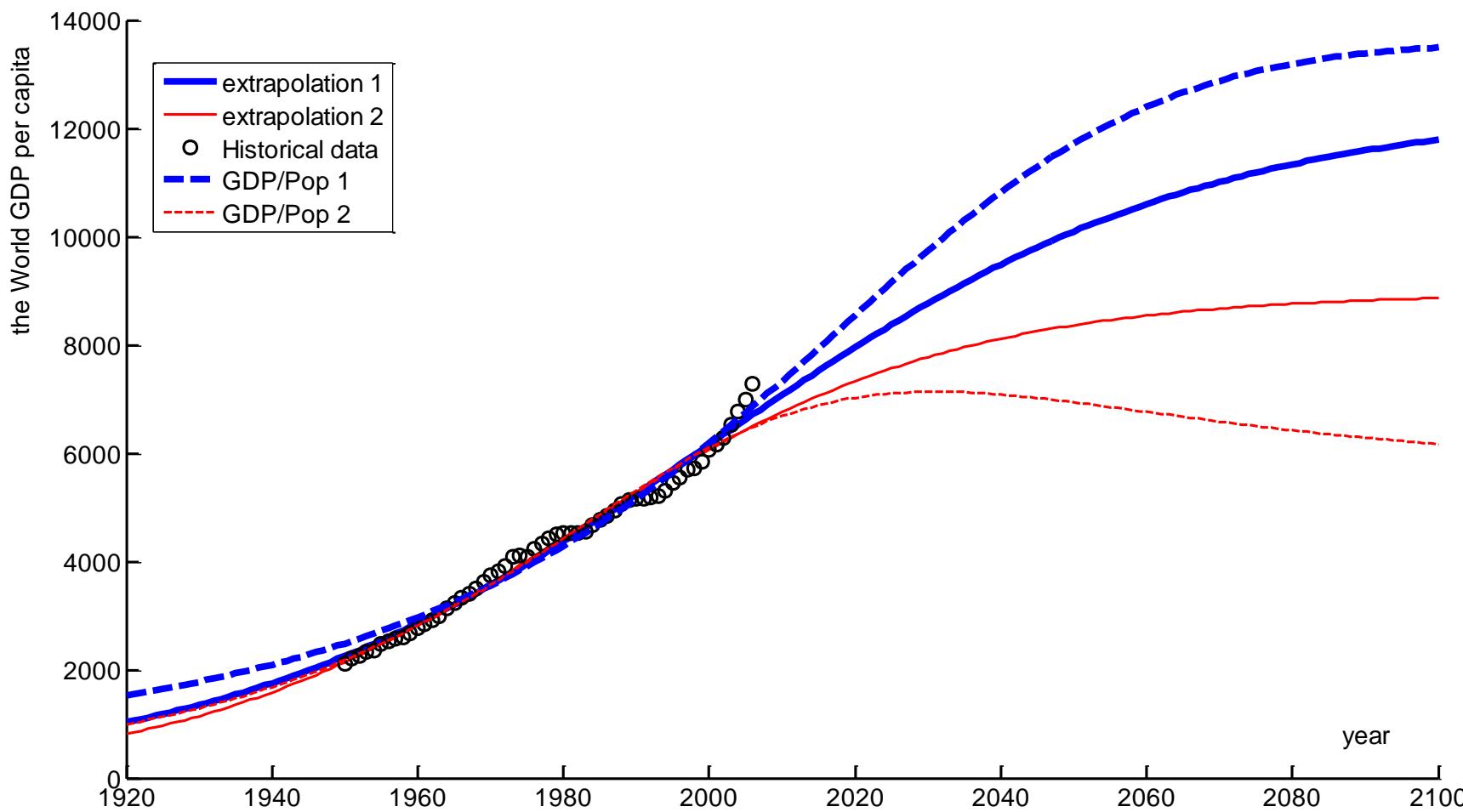


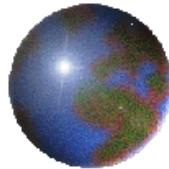
GDP per capita calculated from population and GDP forecasts (both logistic curves)





GDP per capita calculated from population and GDP forecasts (both logistic curves) and forecasted by extrapolation





So far, so good (?)

- GDP Historical data 1980-2006
- Identification (logistic 1)

K	Δt	t_m	Identification error (Distance)
$0.99997717 \cdot 10^{14}$	132.9904	2169.188	$7.125510880 \cdot 10^{11}$
$97.68471900 \cdot 10^{14}$	133.3967	2308.901	$7.103876721 \cdot 10^{11}$
$998.55000000 \cdot 10^{14}$	133.4024	2379.482	$7.103675573 \cdot 10^{11}$
$95917.25000000 \cdot 10^{14}$	133.4024	2518.060	$7.103653992 \cdot 10^{11}$
$9718381.00000000 \cdot 10^{14}$	133.4023	2658.257	$7.103653768 \cdot 10^{11}$
$59807200.00000000 \cdot 10^{14}$	133.4025	2713.420	$7.103653766 \cdot 10^{11}$

- Exponential 1

$Y(1980)$	Growth rate γ	Identification error
$1.9243691 \cdot 10^{13}$	0.032941280	$7.103653765 \cdot 10^{11}$

- Exponential 2

$Y(1980)$	Growth rate γ	Identification error
$1.9516935 \cdot 10^{13}$	0.031941294	0.02033412209

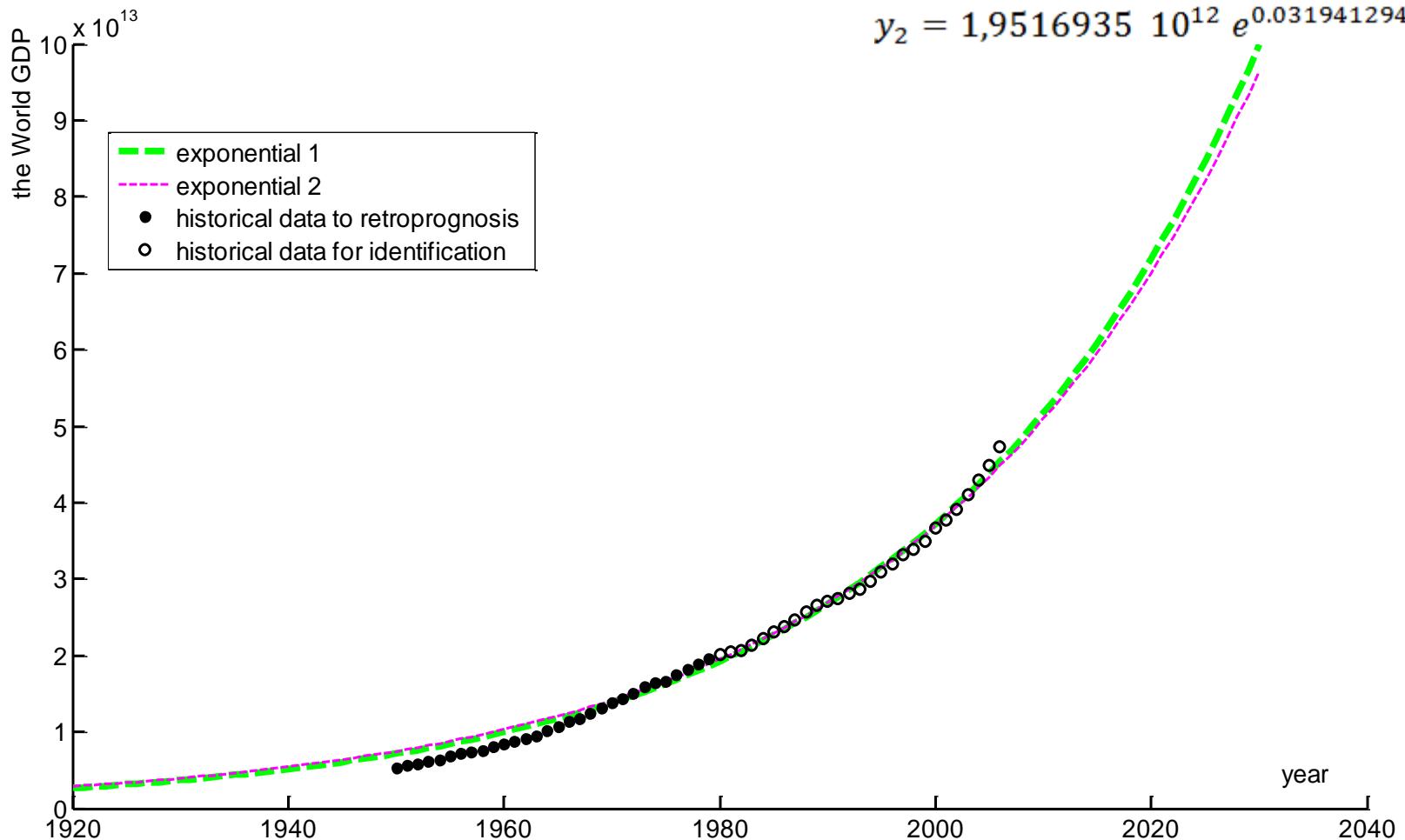


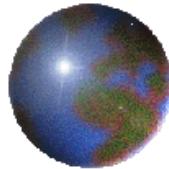
GDP forecast

Identification period 1980-2006

$$y_1 = 1.9243691 \cdot 10^{13} e^{0.03294128(t-1980)}$$

$$y_2 = 1,9516935 \cdot 10^{12} e^{0.031941294(t-1980)}$$





So far, so good (?)

- ◆ GDP Historical data 1950-1971
- ◆ Identification (logistic 1)

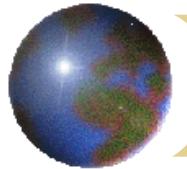
K	Δt	t_m	Identification error (Distance)
$7.8277065 \cdot 10^{15}$	92.9378	2201.748	$1.088235452 \cdot 10^{11}$
$99.9828330 \cdot 10^{15}$	92.9386	2255.624	$1.088209534 \cdot 10^{11}$
$776.2378700 \cdot 10^{15}$	92.9384	2347.665	$1.088207356 \cdot 10^{11}$

- ◆ Exponential 1

$Y(1950)$	Growth rate γ	Identification error
$5.2962144 \cdot 10^{12}$	0.047283458	$1.088207328 \cdot 10^{11}$

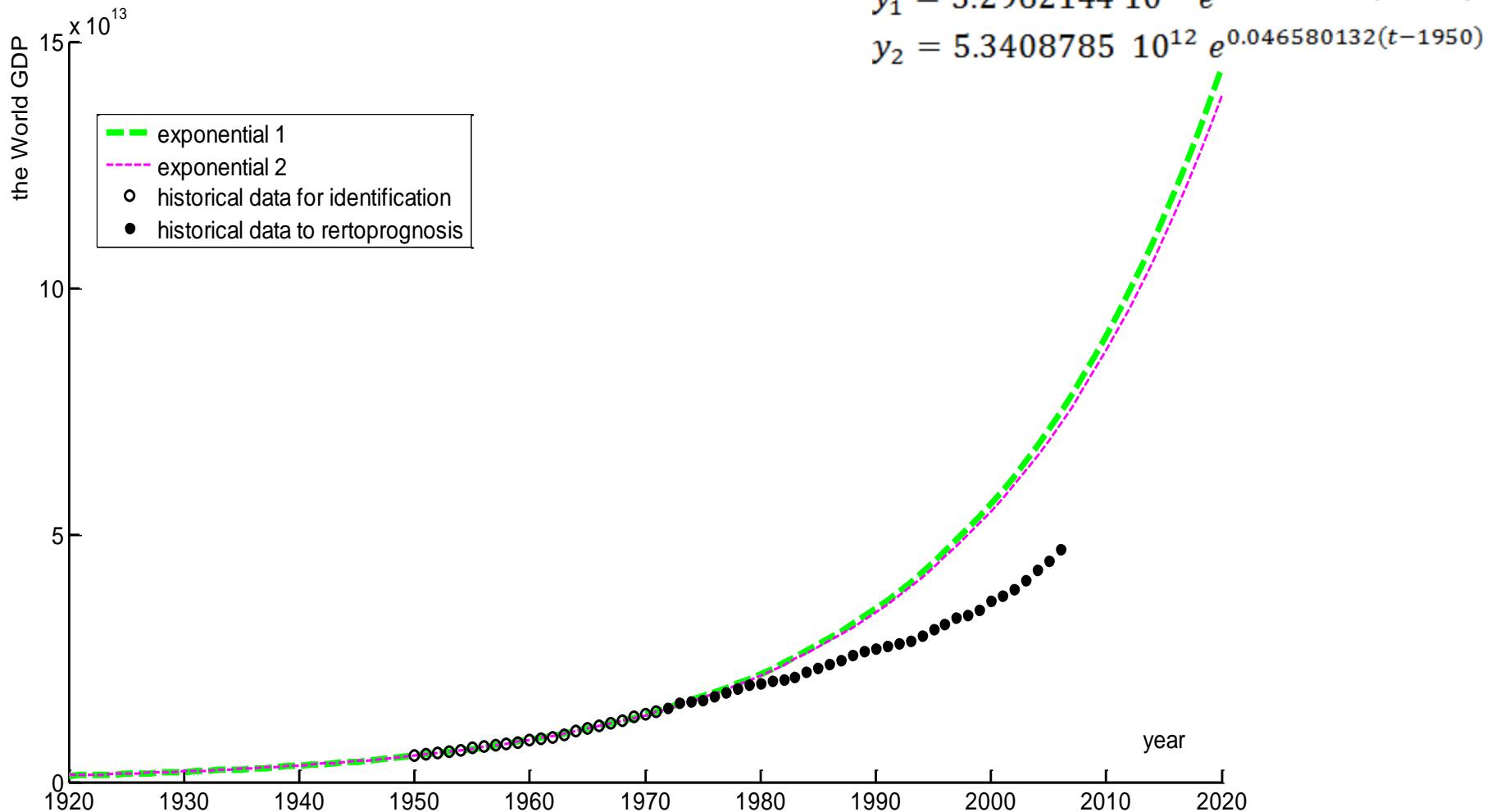
- ◆ Exponential 2

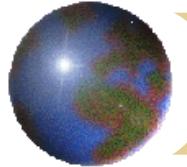
$Y(1950)$	Growth rate γ	Identification error
$5.3408785 \cdot 10^{12}$	0.046580132	0.01219099369



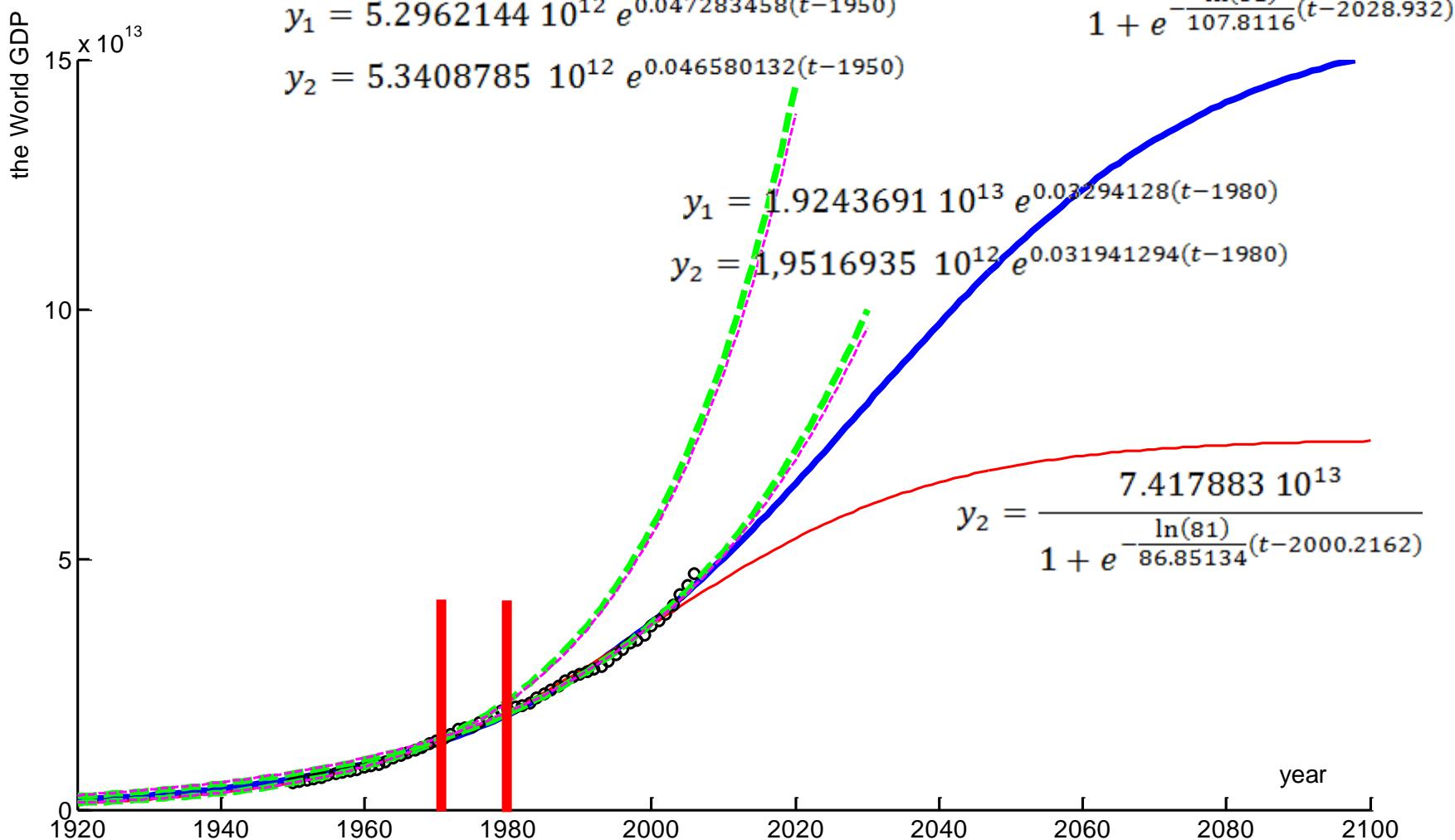
GDP forecast

Identification period 1980-2006





GDP three prognoses





So far, so good (?)

- Population Historical data 1950-1971
- Identification (logistic 1)

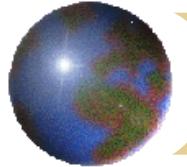
K	Δt	t_m	Identification error (Distance)
$20.515148 \cdot 10^9$	197.2791	2038.442	$1.363592027 \cdot 10^7$
$6287.348200 \cdot 10^9$	232.3073	2363.559	$1.102825711 \cdot 10^7$
$38717522.000000 \cdot 10^9$	232.4238	2825.282	$1.102160683 \cdot 10^7$

- Exponential 1

$Y(1950)$	Growth rate γ	Identification error
$2.516325 \cdot 10^9$	0.018907043	$1.102160577 \cdot 10^7$

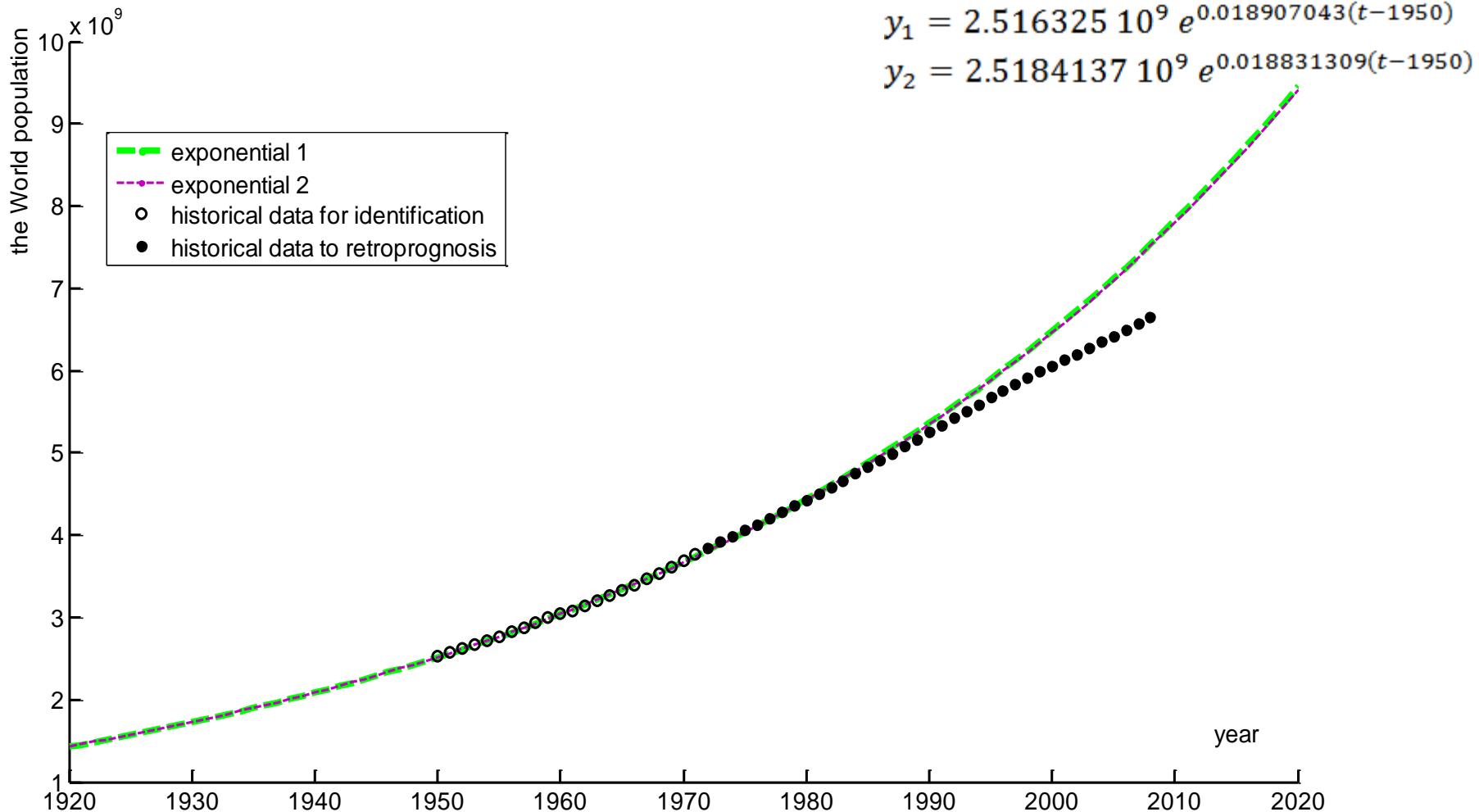
- Exponential 2

$Y(1950)$	Growth rate γ	Identification error
$2.5184137 \cdot 10^9$	0.018831309	0.00339091178



The World population forecast

Identification period 1950-1971





So far, so good (?)

- ❖ Population Historical data 1980-2006
- ❖ Identification (logistic 1)

K	Δt	t_m	Identification error (Distance)
$9.206758 \cdot 10^9$	119.2270	1982.23	$0.7028676 \cdot 10^7$

- ❖ Identification (logistic 2)

K	Δt	t_m	Identification error (Distance)
$9.266125 \cdot 10^9$	120.4273	1982.57	0.001387531253

- ❖ Exponential 1

$\gamma(1980)$	Growth rate γ	Identification error
$4.5238956 \cdot 10^9$	0.014286828	$5,561584868 \cdot 10^7$

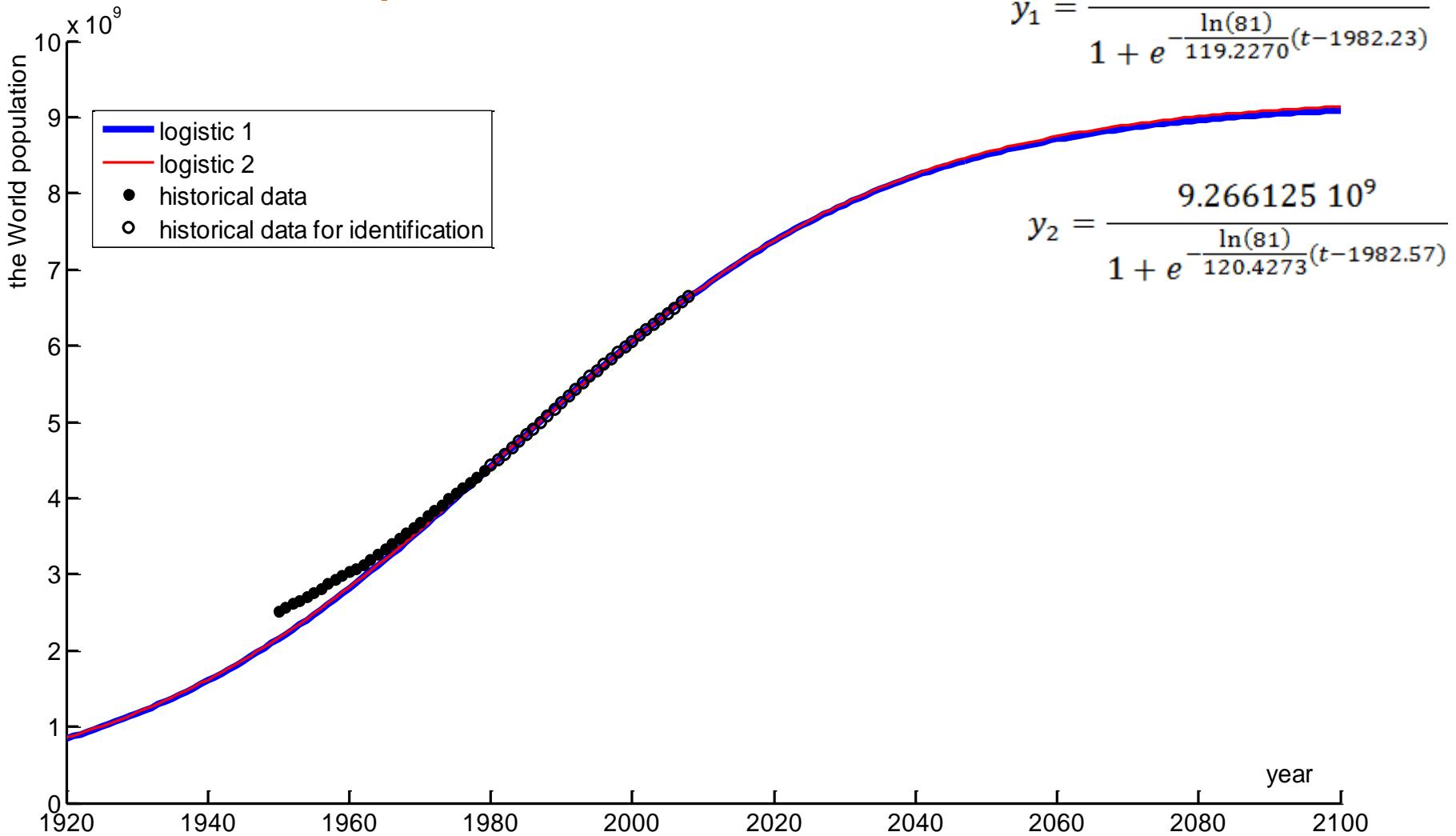
- ❖ Exponential 2

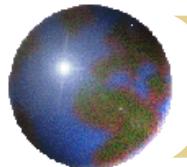
$\gamma(1980)$	Growth rate γ	Identification error
$4.505745 \cdot 10^9$	0.014541383	0.01002837651



The World population forecast

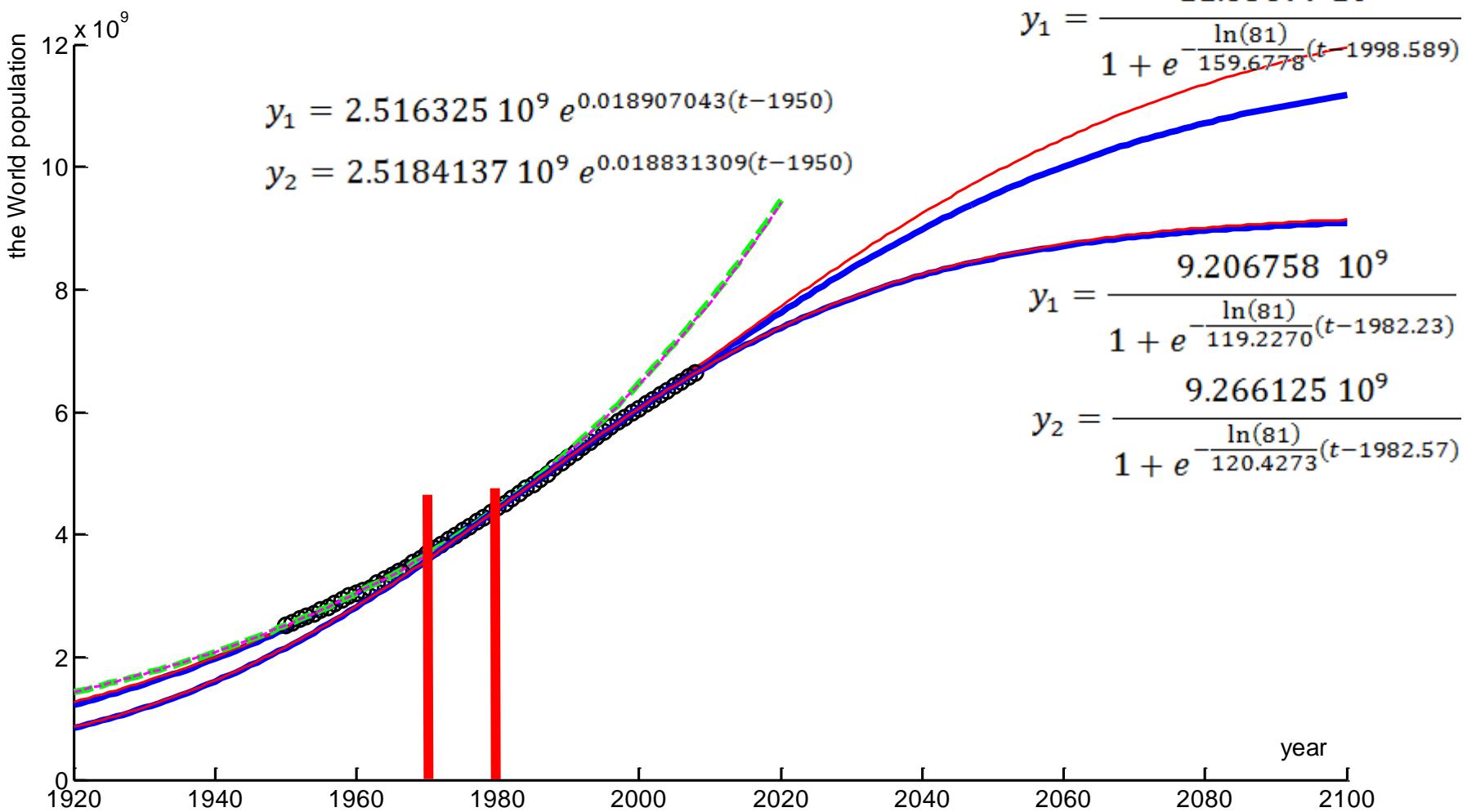
Identification period 1980-2008





The World population forecast

The three prognoses comparison





So far, so good (?)

- ◆ GDP per capita Historical data 1950-1971
- ◆ Identification (logistic 1)

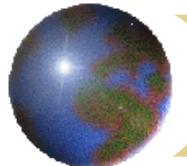
K	Δt	t_m	Identification error (Distance)
899999.82	156.201	2165.102	94.34445365
8999908.40	156.6601	2247.892	94.13893579
38710930.00	156.6988	2299.994	94.12151201

- ◆ Exponential 1

$Y(1950)$	Growth rate γ	Identification error
2114.0825	0.028041745	94.11623718

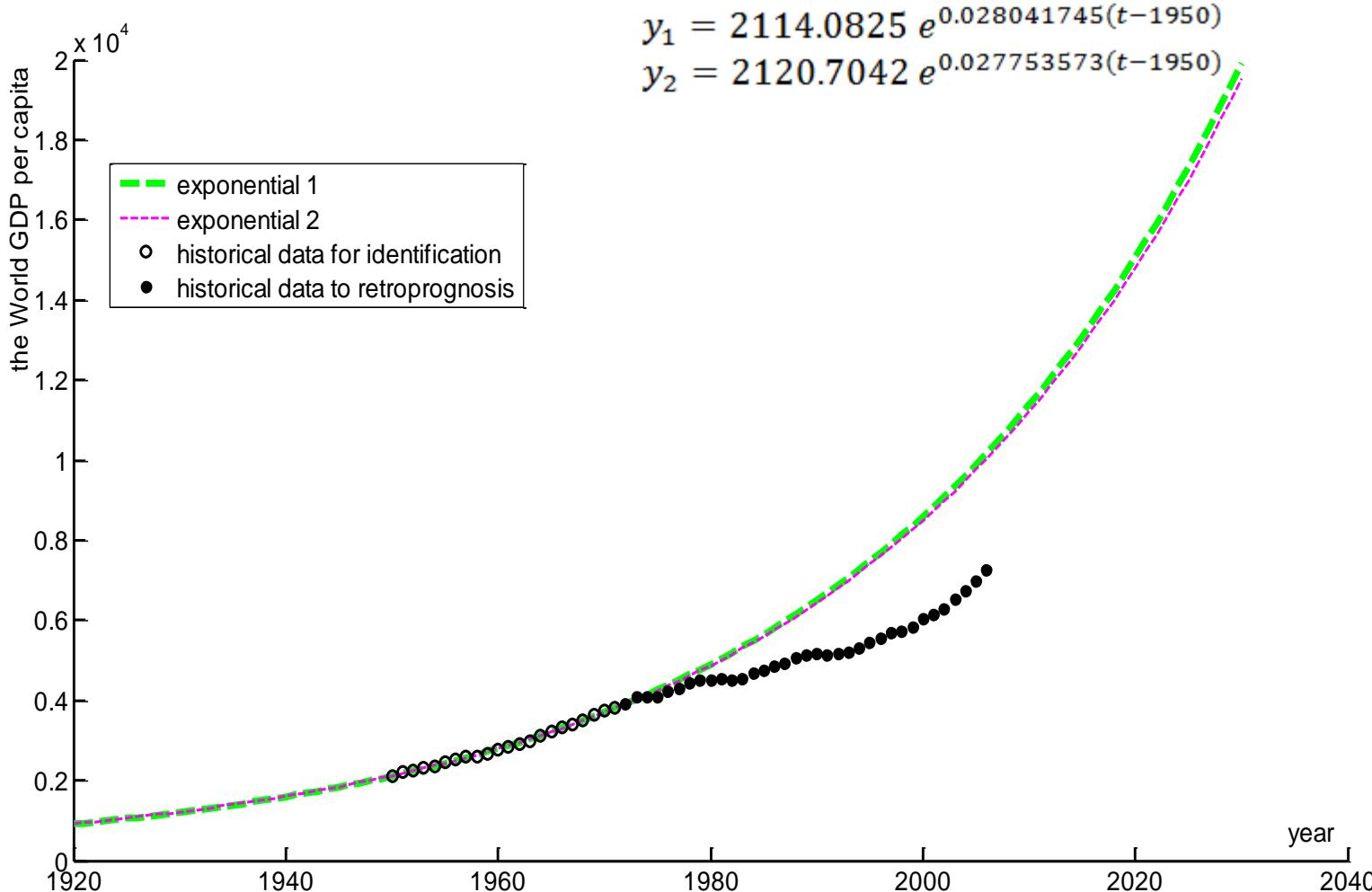
- ◆ Exponential 2

$Y(1950)$	Growth rate γ	Identification error
2120.7042	0.027753573	0.01059338021



The GDP per capita forecast

Identification period 1950-1971





So far, so good (?)

- ◆ GDP per capita Historical data 1980-2006
- ◆ Identification (logistic 1)

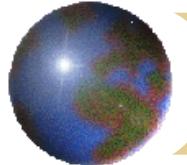
K	Δt	t_m	Identification error (Distance)
18.975492	238.3667	2184.235	538.7584161
9989.583600	245.2912	2541.190	511.3574212
827547.000000	245.3055	2787.785	511.3353354
8005822.300000	245.3055	2914.470	511.3352584

- ◆ Exponential 1

$Y(1980)$	Growth rate γ	Identification error
4297.3220	0.01791420	511.3352495

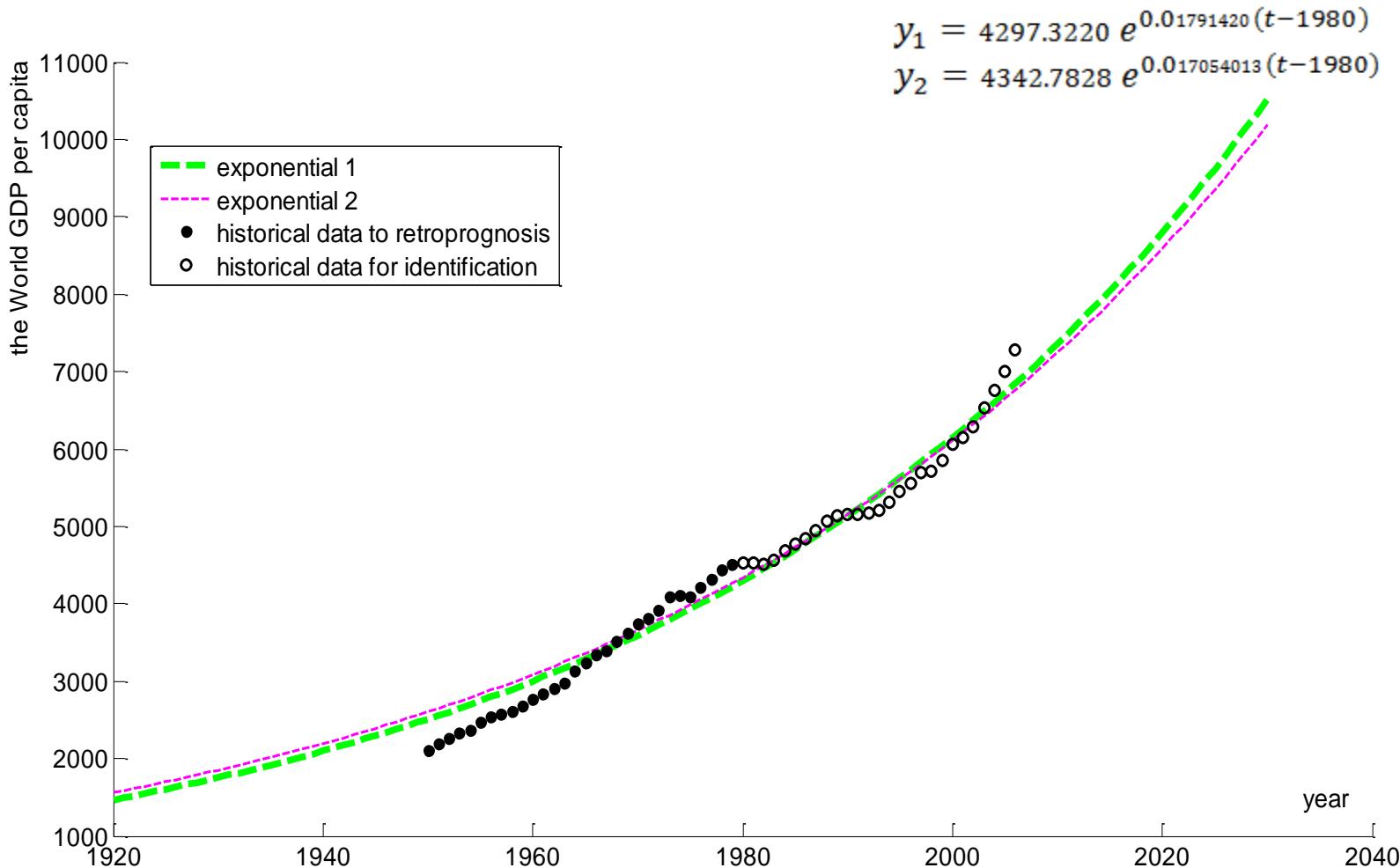
- ◆ Exponential 2

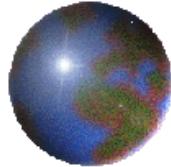
$Y(1980)$	Growth rate γ	Identification error
4342.7828	0.017054013	0.02713080275



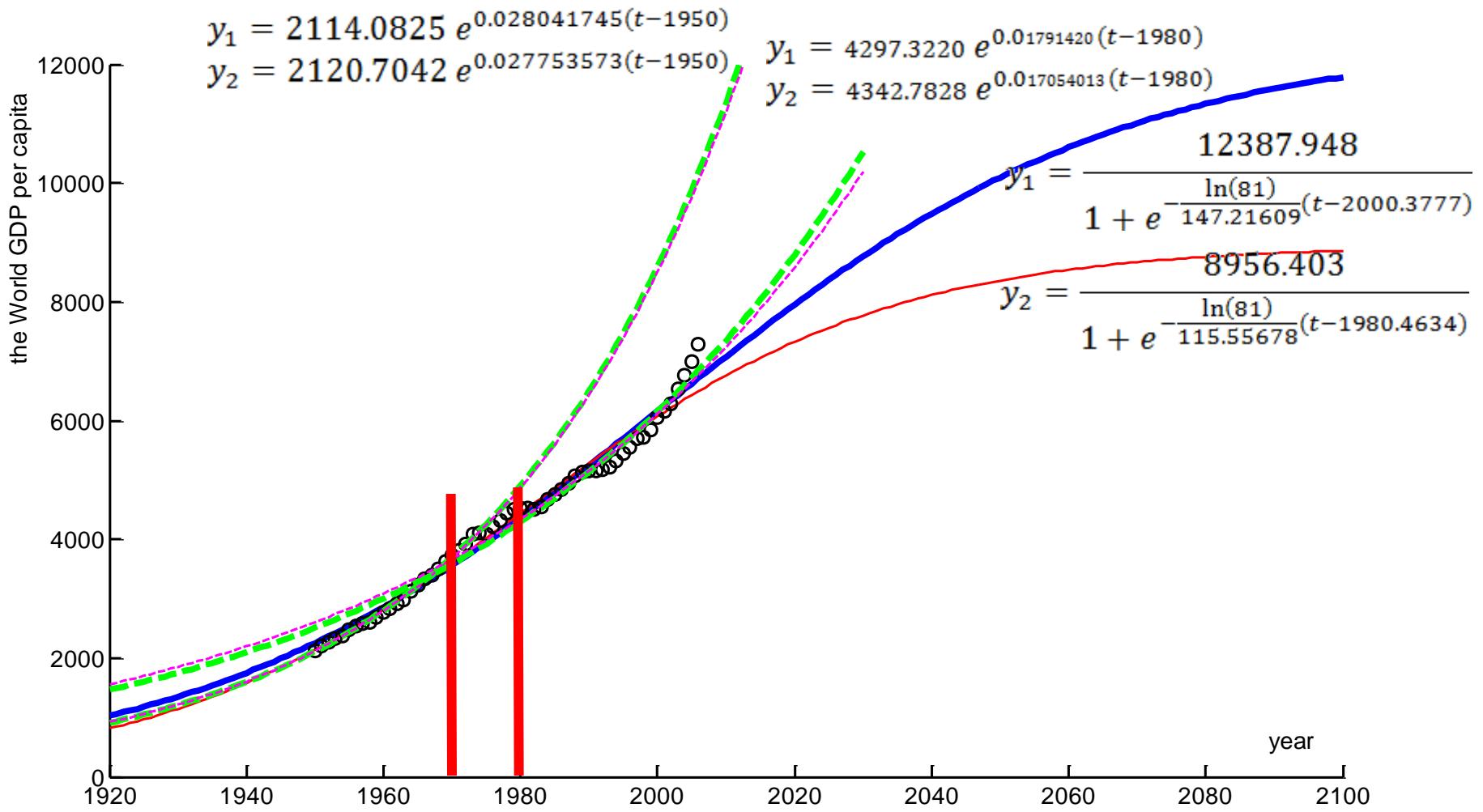
The GDP per capita forecast

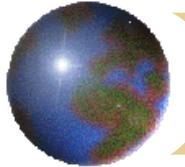
Identification period 1980-2006





GDP per capita – three prognoses





Substitution model

Witold Kwasnicki, Halina Kwasnicka 'Long-Term Diffusion Factors of Technological Development: An Evolutionary Model and Case Study', *Technological Forecasting and Social Change* 52, 31-57, 1996.

- n competing types (e.g., nations)
- $f_i(t)$ – share of type i ,

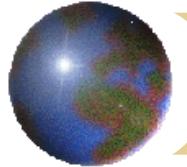
- $c_i(t)$ – competitiveness of type i ,

$$f_i(t+1) = f_i(t) \frac{c_i}{\bar{c}(t)}$$

- $\bar{c}(t)$ - average competitiveness

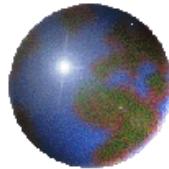
$$\bar{c}(t) = \sum_{i=1}^n f_i(t) c_i(t)$$

- We need to identify c_i and $f_i(t_0)$ for all n types



West, China and the rest of the World

- ◆ West: Austria, Belgium, Cyprus, Denmark, Finland, France, Germany (West Germany from 1950-1988, united Germany from 1989-onwards), Greece, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, Canada, United States, Australia, New Zealand
- ◆ for the China → China and Hong Kong.

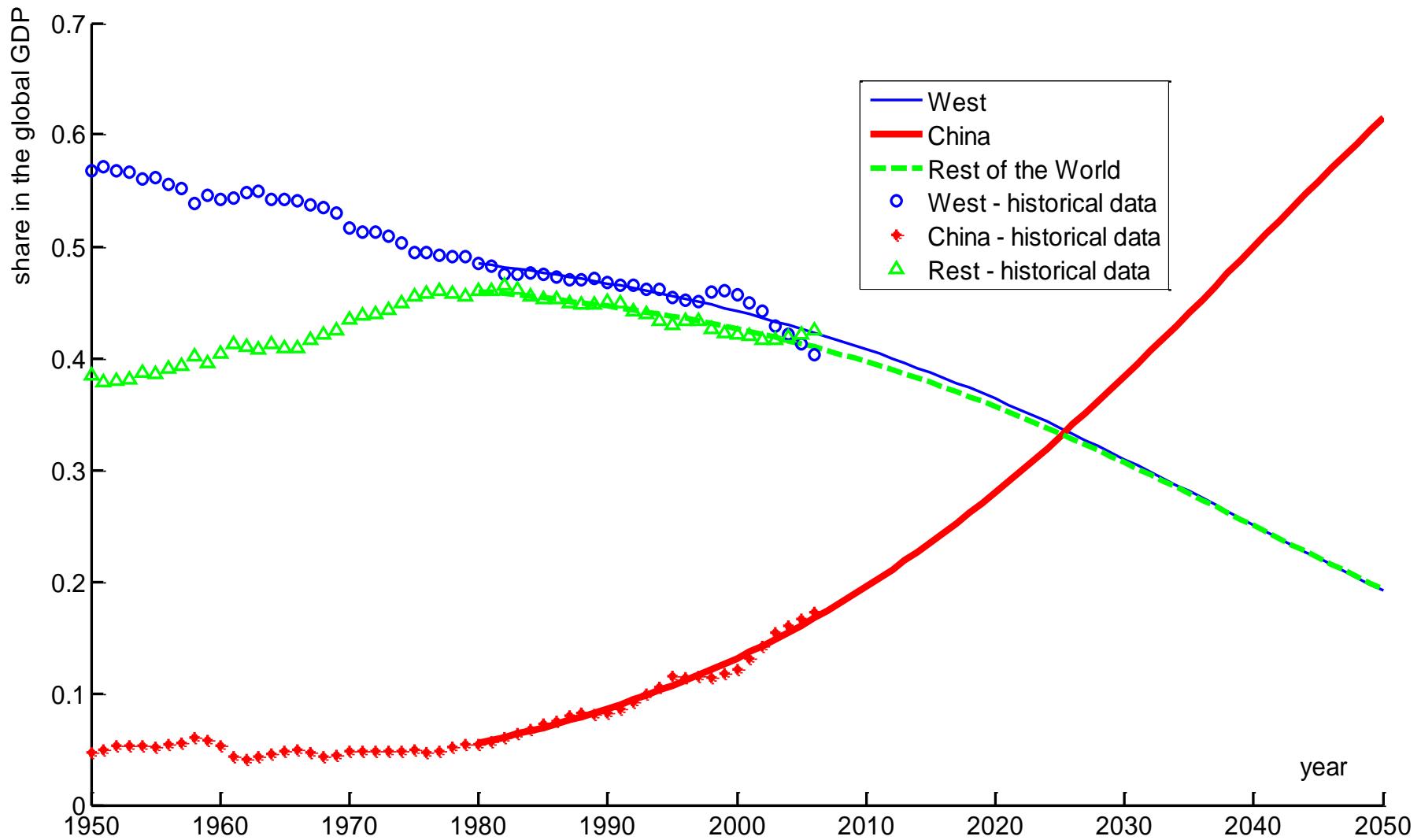


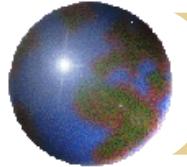
Identification period → 1980 to 2006

	Competitiveness c_i	Share in 1979 $f_k(t_0)$
● West	0.999152	0.486100
● China	1.047807	0.053287
● Rest	1.000000	0.460613



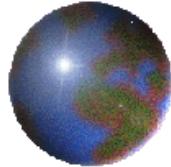
Shares of the three regions



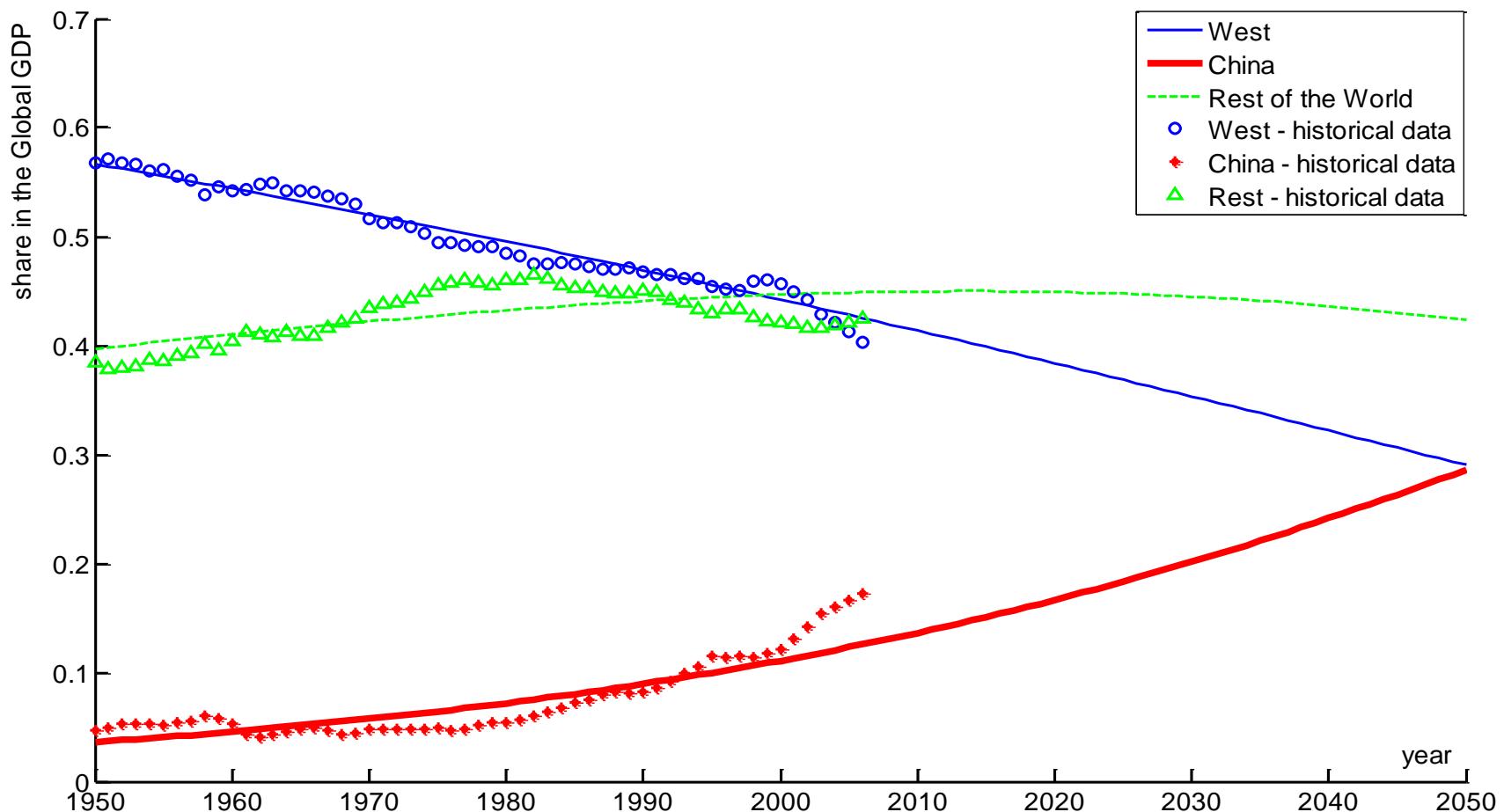


Identification period → 1950 to 2006

	Competitiveness	Share in 1959
● West	0.992706	0.568897
● China	1.020249	0.035354
● Rest	1.000000	0.395749

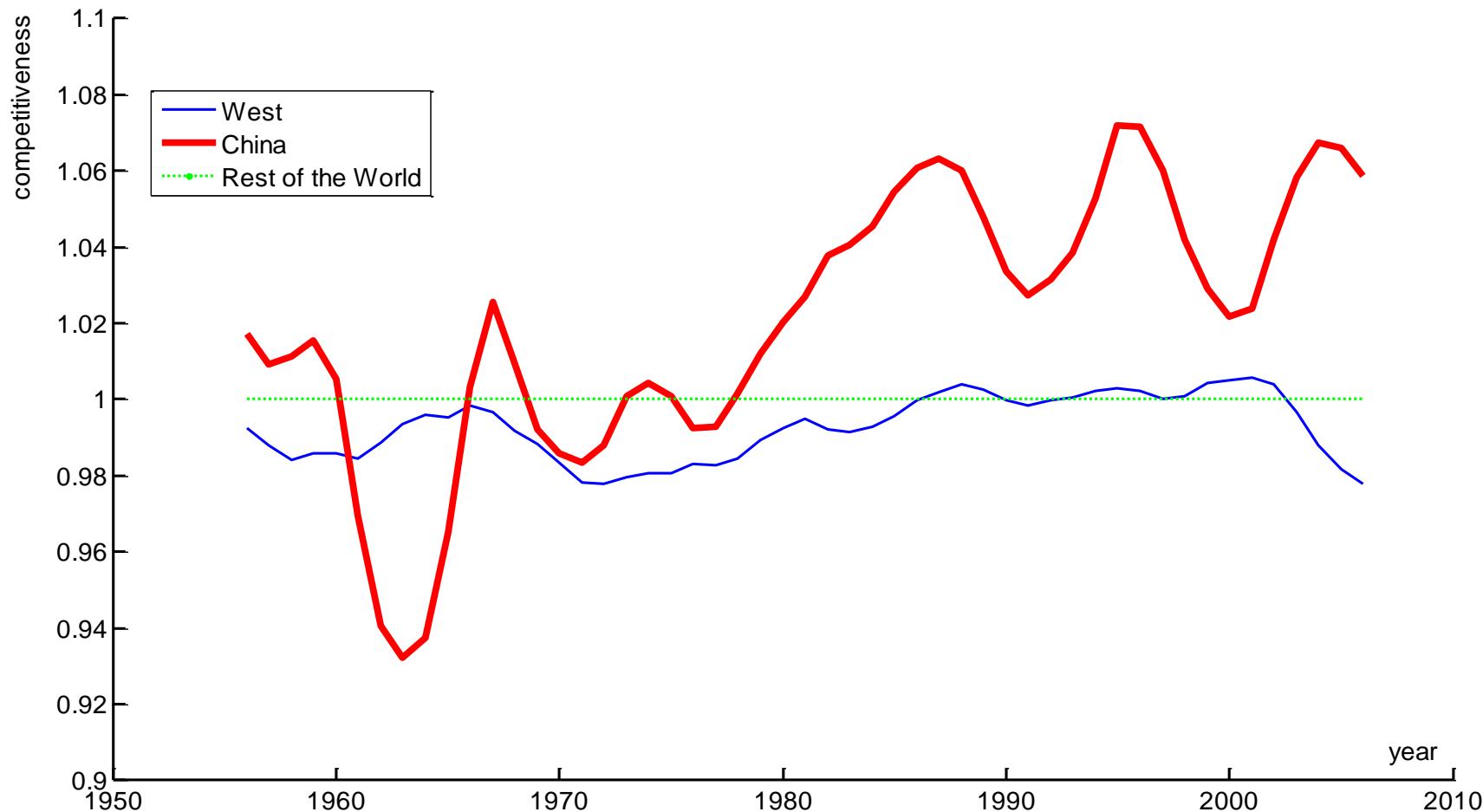


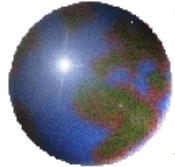
Shares of the three regions





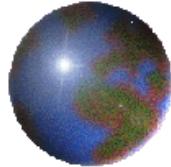
The 7 years moving competitiveness



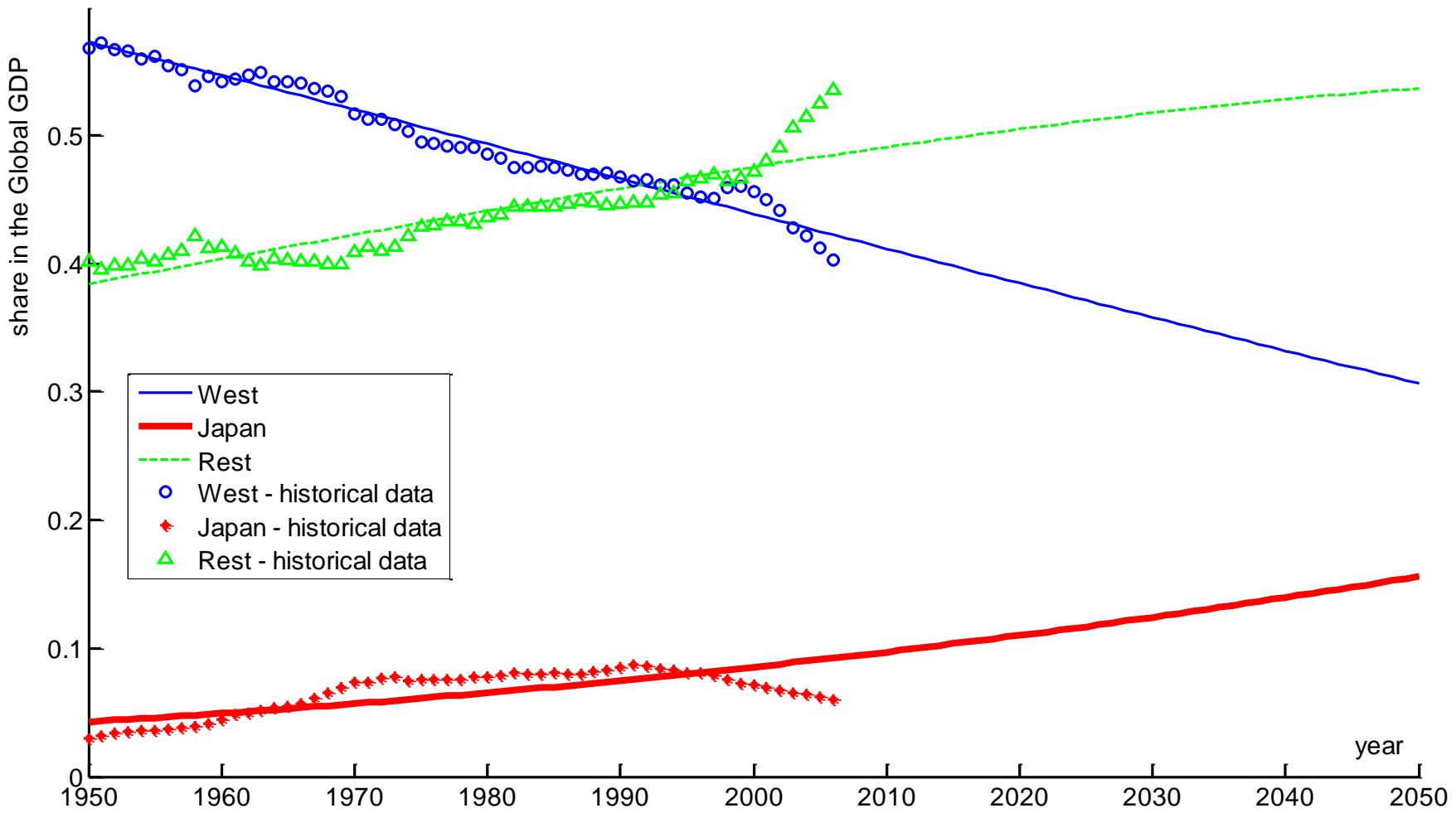


Identification period → 1950 to 2006

	Competitiveness	Share in 1949
● West	0.990449	0.575679
● Japan	1.009686	0.042030
● Rest	1.000000	0.382291

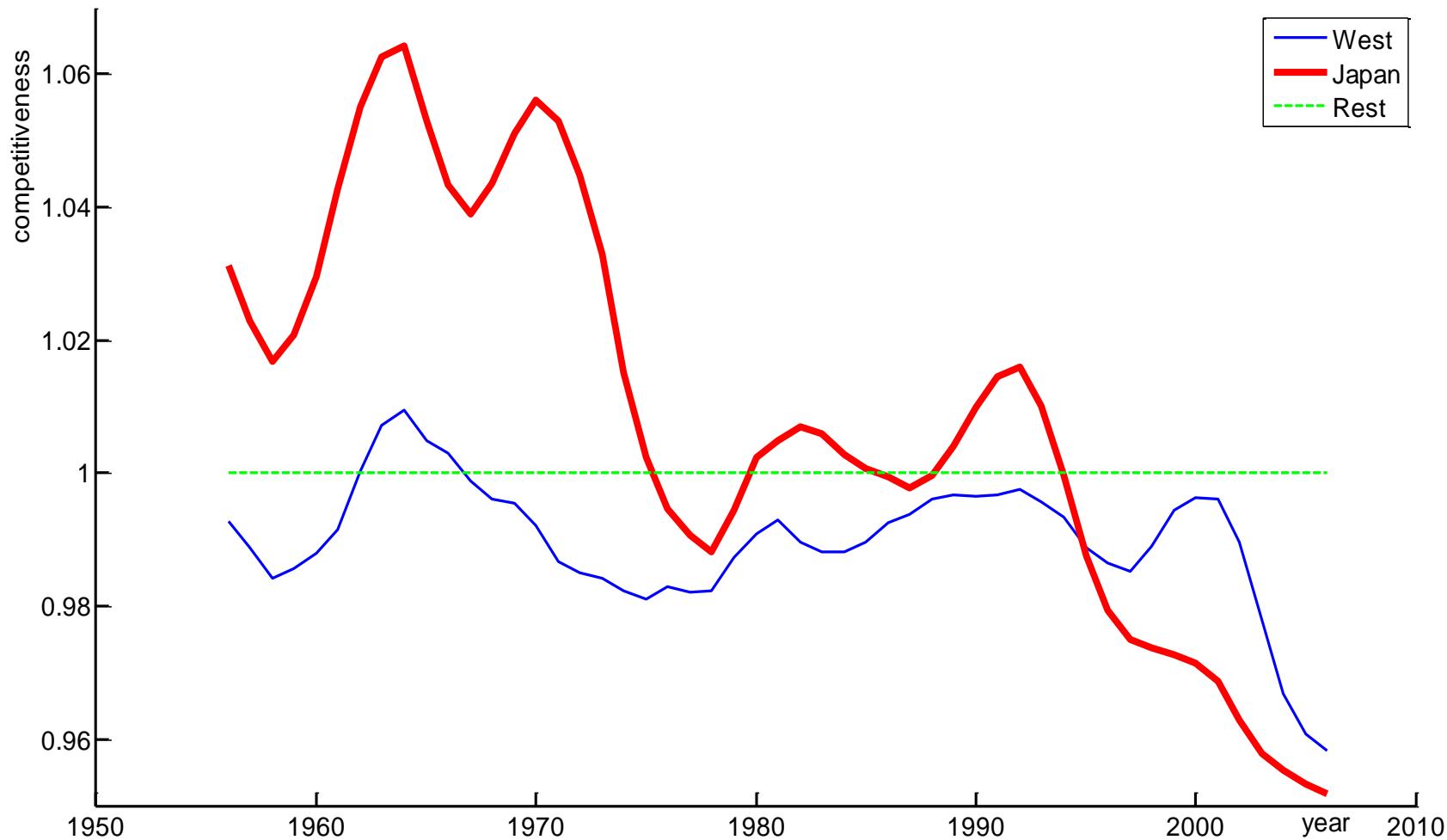


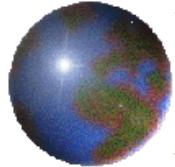
Shares of the three regions





The 7 years moving competitiveness



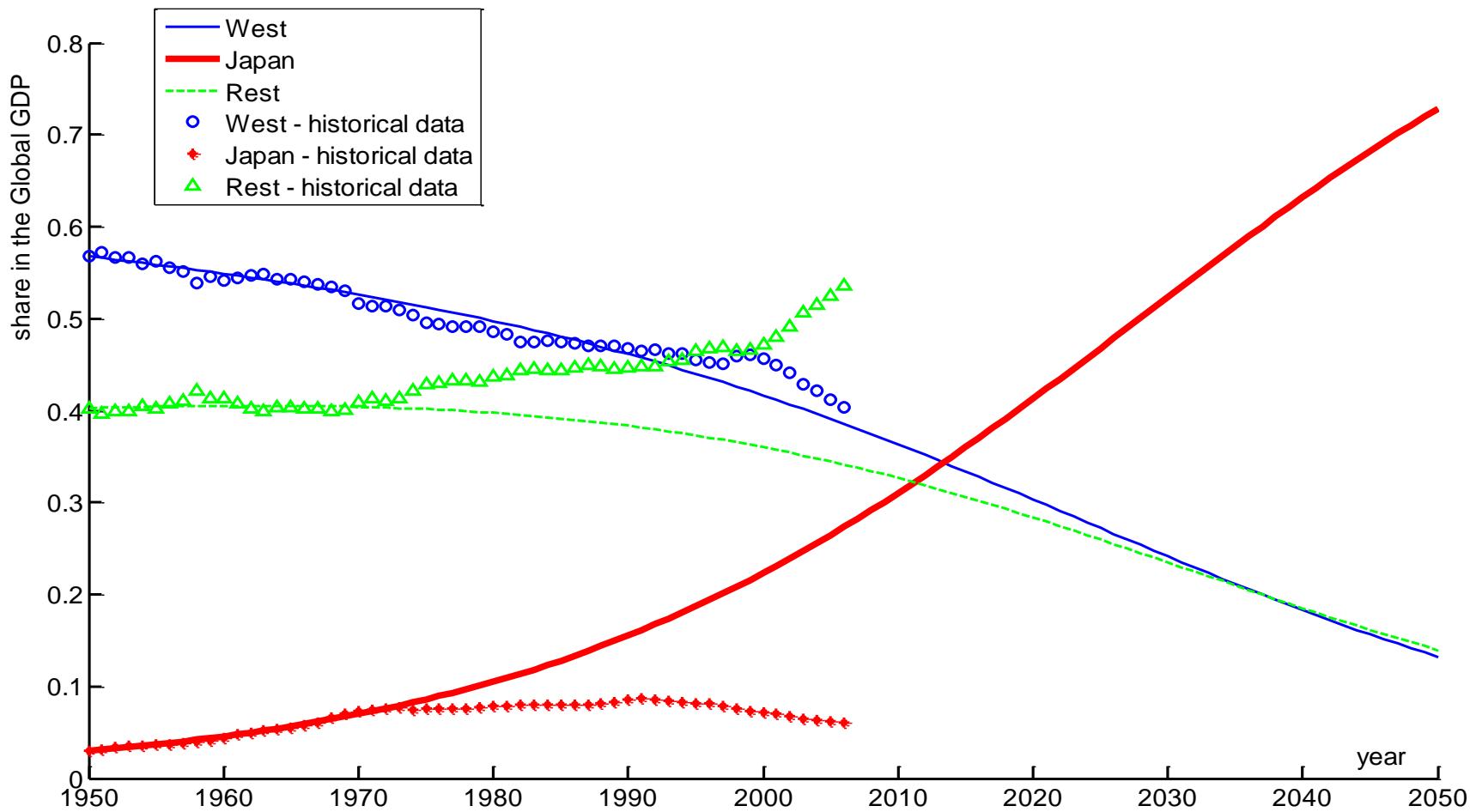


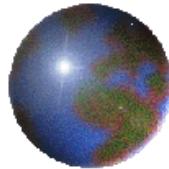
Identification period → 1950 to 1970

	Competitiveness	Share in 1949
● West	0.996064	0.569261
● Japan	1.043551	0.028382
● Rest	1.000000	0.402356



Shares of the three regions

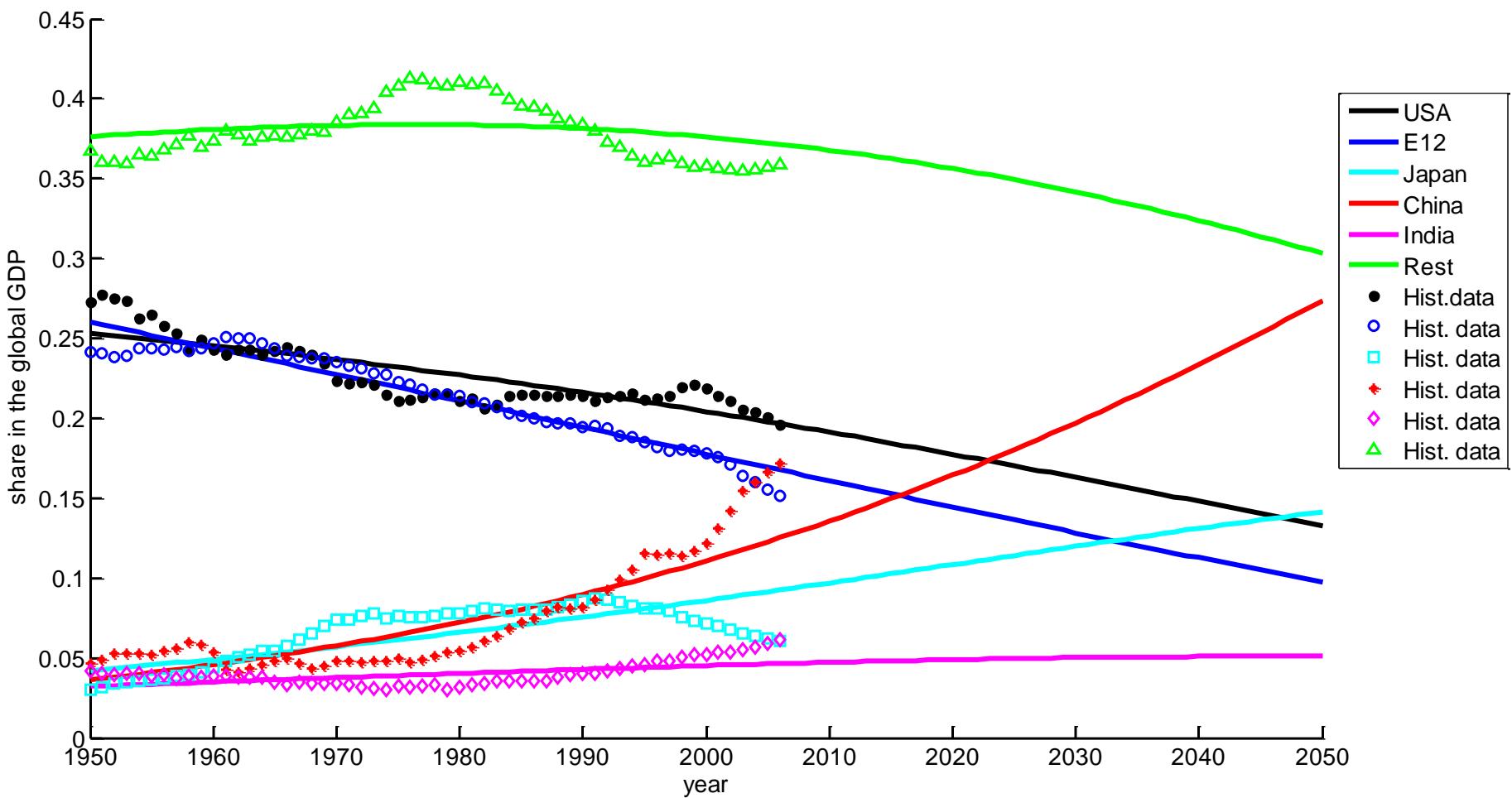
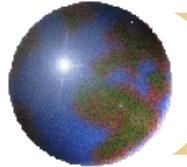




Identification period: 1950 to 2006

	Competitiveness	Share in 1949
● USA	0.995710	0.253936
● E12	0.992412	0.261623
● Japan	1.014378	0.041473
● China	1.022661	0.035302
● India	1.006745	0.032042
● Rest	1.000000	0.375624

- E12 → Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, United Kingdom.





The 14 years moving competitiveness

